THE DUBLIN ACADEMY OF EDUCATION

SUMMER BOOTCAMP

SAMPLE NOTES

MATHS - TRIGONOMETRY

LEAVING CERT (HIGHER LEVEL)

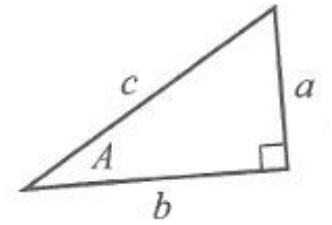
Trigonometry

Trigonometry is worth 7% to 18% of The Leaving Cert.

It appears on Paper 2.

1) Pythagoras

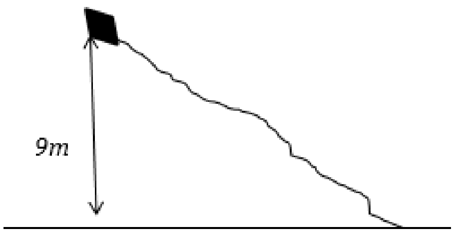
On page 16 of the Formulae and Tables Booklet you will see the following:



$$c^2 = a^2 + b^2$$

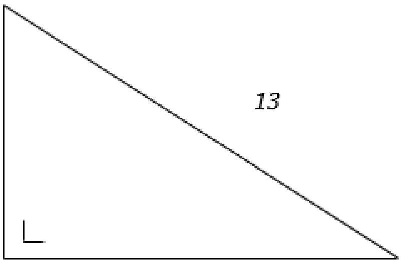
$$(\textbf{longest side})^2 = (\textbf{other side})^2 + (\textbf{other side})^2$$

Example 1:
A kite is held by a string *13m* long. When blown in the wind it is *9m* above the ground as shown in the diagram:



How far is the point A from B, correct to the nearest metre?
List any assumptions you have made.

Solution:

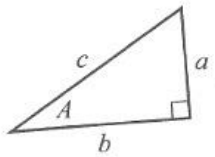


$$13^2 = 9^2 + x^2$$
$$169 = 81 + x^2$$
$$169 - 81 = x^2$$
$$88 = x^2$$
$$\sqrt{88} = x^2$$
$$9.38 = x$$
$$x = 9\ meters$$

Assumptions:
I assumed the string is straight.
I assumed the ground is level.

2) Sin, Cos, Tan

On page 16 of The Formulae and Tables Booklet you will see the following:



$\sin A = \frac{a}{c}$ $\qquad \cos A = \frac{b}{c}$ $\qquad \tan A = \frac{a}{b}$

What this really means is:

$$Sin\ A = \frac{opposite}{hypotenuse}$$

$$Cos\ A = \frac{adjacent}{hypotenuse}$$

$$Tan\ A = \frac{opposite}{adjacent}$$

The way to remember this is:

SOH CAH TOA

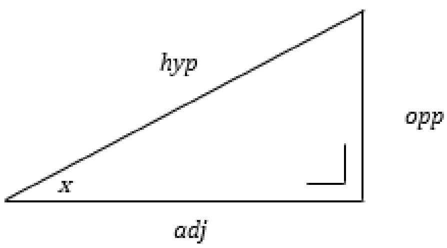
or

Silly Old Harry, Caught A Herring, Trawling Off America

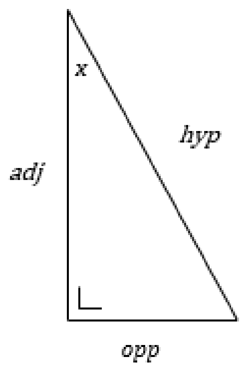
or

Some Old Horses, Can Always Hear, Their Owners Approaching

Labelling the sides:



or



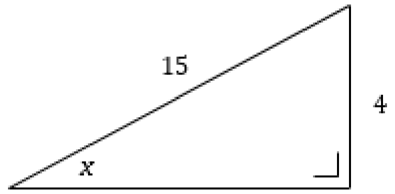
i.e. The 'hyp' is the side across from the right angle.

The 'opp' is the side across from the angle in question.

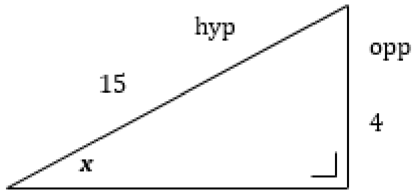
The 'adj' is the other side.

Note: Don't use the letter 'h' for hypotenuse as you may get it mixed up with 'h' for height when you're under pressure in the exam.

Example 1:
Find the measure of the angle *'x'*. Give your answer correct to two decimal places.



Solution:



The sides in question are 'hyp' and 'opp', so we will use *Sin:*

$Sin\ x = \frac{opp}{hyp}$

$Sin\ x = \frac{4}{15}$

$x = Sin^{-1}(\frac{4}{15})$

$= 15.47^{\circ}$

Note: You must be able to change angles from decimals to a thing called 'minutes'.

Example 2:
Change 37.36° to the nearest minute.

Casio Calculator:	To change from decimals to minutes, type in the number with the decimal, then press the button with all the commas (2 buttons above number 8), then press equals.
Sharp Calculator:	To change from decimals to minutes, type in the number with the decimal, then press 2nd function, then press DMS.

You will get 37°21′36"
Now the 37 is the degrees, the 21 is the minutes and the 36 is a thing called seconds. If your value for seconds is 30 or greater you round up. If it is less than 30 you round down.

Therefore we will round up:
= 37°22

Question 2.1

Change 50.27° to the nearest minute.

Question 2.2

Change 50.38° to the nearest minute.

Sometimes an angle will be given to you in degrees and minutes so you have to be careful when putting this into your calculator.

Example 3:
Find $Sin31°26'$ correct to 4 decimal places.

Casio Calculator:	Press Sin, then 31, then the comma button, then 26, then the comma button again, then equals.
Sharp Calculator:	Press Sin, then 31, then the DMS button, then 26, then the DMS button again.

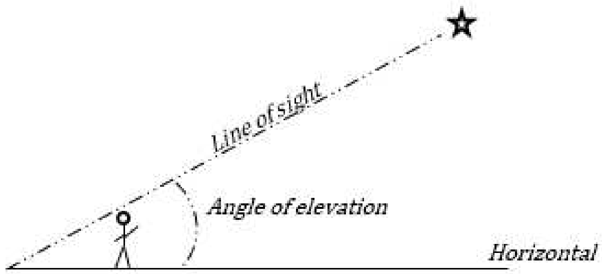
You will get 0.5215061194
(depending on your calculator you may even get more decimal places)
We were asked to round off to 4 decimal places so our answer is 0.5215

Question 2.3

Evaluate $sin35°7'$ to four decimal places.

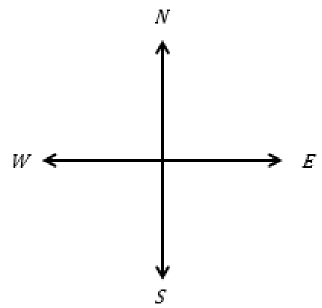
There are three other points we must know to help us with the 'real life' scenarios which are at the heart of Project Maths:

i) Angle of elevation:
This is the angle your line of vision makes with the horizontal when looking up:

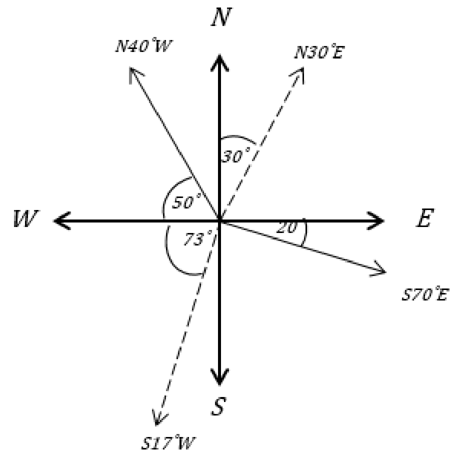


ii) Angle of depression:
This is the angle your line of vision makes with the horizontal when looking down:

iii) Compass directions:



For example:

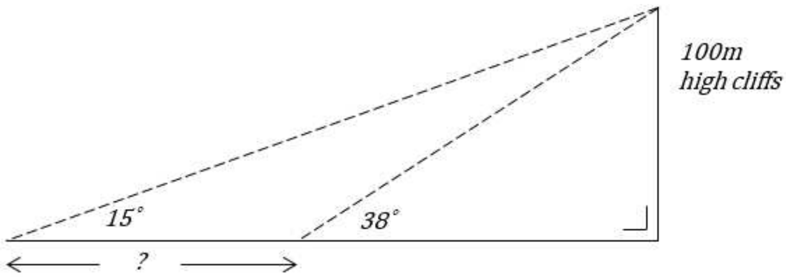


Example 4:

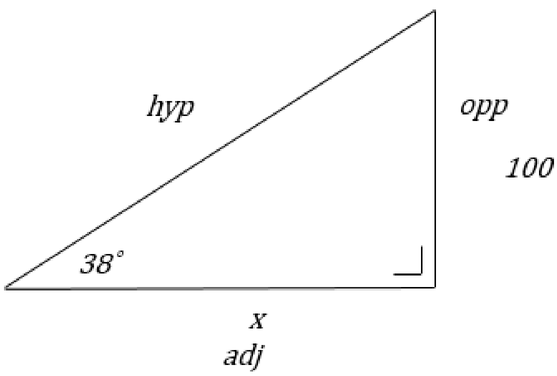
A boat is anchored off the coast near *100m* high cliffs in Co. Kerry. The angle of elevation from the boat to the cliff top is *15˚*. The next day the boat moves further towards land and anchors again. This time the angle of elevation is *38˚*. How far did the boat move?

Give your answer correct to the nearest metre and make assumptions as necessary, stating them clearly.

Solution:



Take out the triangle on the right:

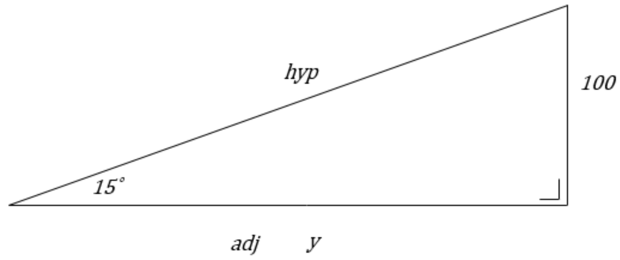


$$Tan\ 38^\circ = \frac{100}{x}$$

$$x = \frac{100}{Tan\ 38^\circ}$$

$$x = 127.994m$$

Now take out the overall triangle:



$Tan\ 15^\circ = \frac{100}{y}$

$y = \frac{100}{Tan\ 15^\circ}$

$y = 373.205m$

The distance the boat moves is y – x:

$= 373.205 - 127.994$

$= 245.211$ *metres*

≈ 245 *metres*

Assumptions: It was assumed the cliffs were vertical, thus giving a right angle where they meet at the sea.

Question 2.4

The beach at Brittas Bay slopes downwards at a constant angle of 12˚ to the horizontal. How far out horizontally into the sea can a man walk before the water covers his head? Make assumptions as necessary.

3) Radians

Just as distance can be measured in miles or kilometers, (1km ≈0.625 miles) angles can be measured in degrees or radians. (1 radian ≈ 57˚)

You must remember:

$1^\circ = \frac{\pi}{180}$ *radians* π *radians* = $180°$

Note: This π isn't equal to 3.14 like it is when we are dealing with circles, so don't get them mixed up!!

Example 1:

Convert 40˚ to radians

$$1^\circ = \frac{\pi}{180} \text{ radians}$$
$$=> 40^\circ = 40\left(\frac{\pi}{180}\right) \text{ radians}$$
$$= \frac{40\pi}{180} \text{ radians}$$
$$= \frac{2\pi}{9} \text{ radians}$$

Example 2:

Convert $\frac{5\pi}{9}$ radians to degrees

$$\pi \text{ radians} = 180$$
$$=> \frac{5\pi}{9} \text{ radians} = \frac{5}{9}(180^\circ)$$
$$= 100^\circ$$

Question 3.1

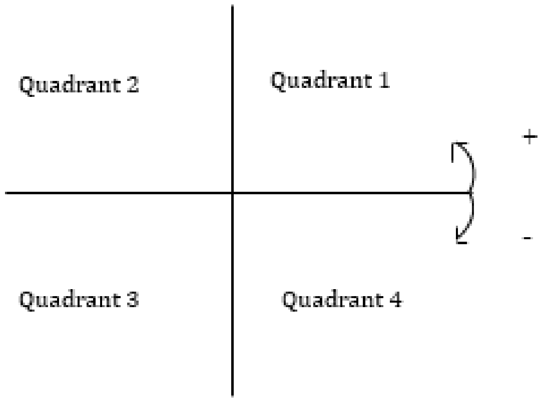
(i) Convert 60˚ to radians

(ii) Convert $\frac{\pi}{4}$ radians to degrees

(iii) How many radians are there in a full rotation?

4) Angles of greater than 360°(or 2π radians)

In trigonometry we have four quadrants:



- For example: 370° would do a full rotation plus another 10°. It terminates (or stops) in Quadrant 1
- 210° would terminate in Quadrant 3
- -60° would terminate in Quadrant 4
- -480° would terminate in Quadrant 3
- $\frac{6\pi}{5}$ radians would terminate in Quadrant 3

Question 4.1

In which quadrant would the following angles terminate?

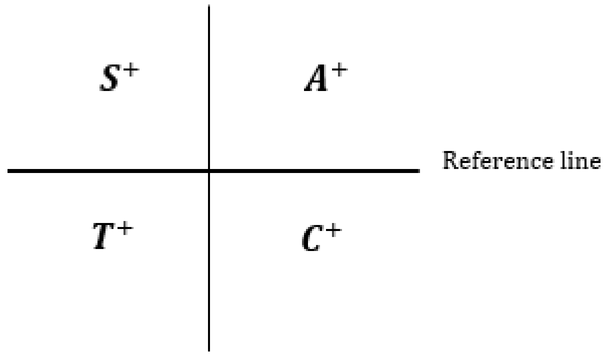
(i) -880°

(ii) $\frac{14\pi}{3}$ radians

(iii) $-\frac{17\pi}{7}$ radians

5) Trigonometric Equations

Consider the following diagram (<u>S</u>ex <u>A</u>nd <u>T</u>he <u>C</u>ity):



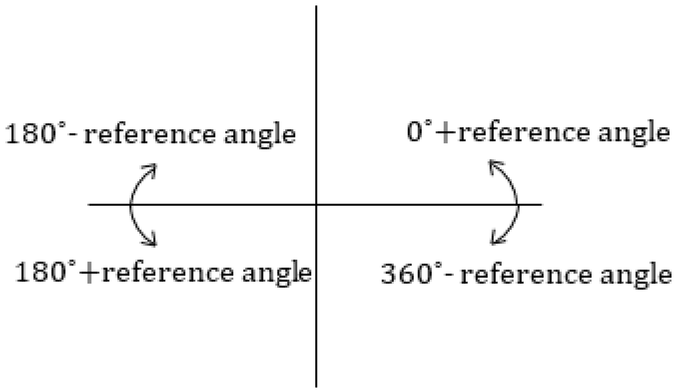
$\boldsymbol{A}^{+}$stands for 'All plus'

$\boldsymbol{S}^{+}$stands for 'Sin plus'

$\boldsymbol{T}^{+}$stands for 'Tan plus'

$\boldsymbol{C}^{+}$stands for 'Cos plus'

To read angles in any quadrant we use the following diagram:

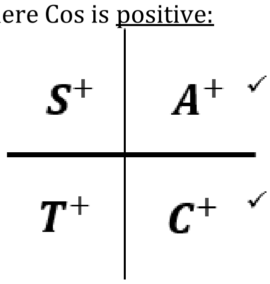


To solve trigonometric equations we will use these 2 diagrams. These equations are best explained with some examples.

<u>Example 1:</u>
Solve for θ: Cos θ = $\frac{1}{\sqrt{2}}$, 0≤ θ≤360˚

Step 1: Find the reference angle by finding the inverse of the value given:
=> Reference angle $= Cos^{-1}(\frac{1}{\sqrt{2}})$
$= 45°$

Step 2: Use the SATC diagram to establish where Cos is <u>positive:</u>



Step 3: Use the reference diagram:
θ = 0˚ + 45˚ or θ = 360˚ - 45˚
=45˚ = 315˚

Question 5.1

Solve for θ: $sin\theta = \frac{\sqrt{3}}{2}$, $0° \leq \theta \leq 360°$

Next let's try one with a minus sign:

Example 2:

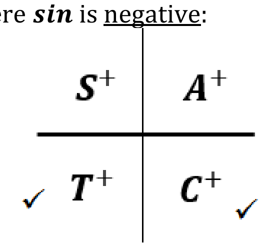
Solve for θ: Sin θ = $-\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$ 0°≤ θ≤360°

Step 1: Find the reference angle by finding the inverse of the value given and by ignoring the minus sign.

Reference angle = $\boldsymbol{Sin^{-1}}(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}})$

= 45°

Step 2: Use the SATC diagram to establish where $\boldsymbol{sin}$ is <u>negative</u>:



Step 3: Use the reference diagram

θ = 180° + 45° or θ = 360° – 45°

= 225° = 315°

Question 5.2

Solve for θ: $cos\theta = -\frac{1}{\sqrt{2}}$, $0° \leq \theta \leq 360°$.

Question 5.3

Find correct to one decimal place, two values of A for $CosA = -\frac{3}{5}$, where $0° \leq A \leq 360°$.

There may also be a bit of rearranging to be done before starting the question. Let's do an example to show this…

Question 5.4

Solve, correct to one decimal place:

$7Cos\ \theta + 3 = 0$ where $0° \leq \theta \leq 360°$

The question can also be given in radians rather than degrees. You should do the questions in degrees, because radians are a load of crap. Then just change to radians at the end to get the full marks ☺.

Question 5.5

Solve for θ:

Tan $\theta = \frac{1}{\sqrt{3}}$, $0 \leq \theta \leq 2\pi$

Question 5.6

Solve for θ: (θ in radians) if Cos $\theta = -\frac{1}{\sqrt{2}}$, $0 \leq \theta \leq 2\pi$

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